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GREY MARKOV (1,1) MODEL FOR FORECASTING THE PERCENTAGE OF THE POPULATION THAT EXPERIENCED HEALTH COMPLAINTS IN INDONESIA

NUR'AINUL MIFTAHUL HUDA[a], NURFITRI IMRO'AH[b]*

[a] *Department of Mathematics, Universitas Tanjungpura, Pontianak, Indonesia,*
[b] *Department of Statistics, Universitas Tanjungpura, Pontianak, Indonesia,*
email : nur'ainul@fmipa.untan.ac.id, nurfitriimroah@math.untan.ac.id

Diterima Direvisi Dipublikasikan

Abstrak. In mathematics, in addition to the time series model, Autoregressive, Moving Average, or Autoregressive Moving Average, the Grey-Markov (1,1) model can be employed for forecasting. One of the gains of this model is that it may cover a minimum quantity of data, which is beneficial in situations when the amount of data that is available is restricted but is not excessively vast. This model works well with data that does not exhibit a great deal of variability. The Grey model was further developed into the Grey-Markov model by including the idea of a Markov chain into the original model. In this particular investigation, the processes consist of first forming a sequence using a 1-Accumulated Generating Operation (1-AGO), then forming a sequence using an MGO, and finally predicting using an AGO. The procedure that came before it is actually a modeling procedure for the Grey model. In addition, in order to model Grey-Markov(1,1), it is necessary to initially compute the relative inaccuracy of the forecast that came before it. The following step is to partition the outcome of the relative error into numerous states, one for each interval of the relative error. After that, each error is categorized based on a state that has been specified in advance. The state that is defined within the class is used as the basis for making predictions. The percentage of the population in Indonesia that reports having health difficulties on a yearly basis was chosen as the case study for this research because it is relevant to the topic at hand. The data came from the Central Statistics Agency in the United Kingdom. The period covered by the data is from 1996 to 2021. The purpose of this research is to investigate the structure of the Grey-Markov Model (1,1) and provide a forecast regarding the proportion of the general population that will be affected by health issues in the year 2022. According to the findings of this research project, the forecast of the proportion of the population in Indonesia that suffered health complaints in 2022 produced predictive data that was 30.36%, with a very good accuracy value of 2.43%.

Kata Kunci: Grey, Markov, State, Forecasting

1. Introduction

In 1982, Deng Julong was the one who first recognized System Grey theory, often known as the Grey forecasting model. According to Julong, System Grey can

*Nurfitri Imro'ah

be utilized if there is a deficiency of information within a data set, including the absence of a structure, a restricted quantity of data, or an operating method [1]. The Grey System was eventually developed into the Markov Chain analysis, which led to the creation of the Grey-Markov model (1,1). The Grey-Markov (1,1) model incorporates the idea of state transitions from one point in time to the next on data that is ambiguous. There are a number of unknowns, which lend credence to the idea of including Markov Chain analysis into the Grey Model. The Grey Model can be expressed in its general form as (d, p), where d is the number of differentials that were calculated and p indicates the number of variables [2]. The Grey Model (1,1) was selected for use in this investigation since the process of differentiation was carried out just once, and the number of variables that were factored into each prediction consisted of a single factor. The Grey-Markov model (1,1) is the main model used in Grey's prediction theory. It is the Grey Model with a single variable of order one, and it is constructed from multiple data while still generating answers with high precision. Grey's prediction theory was developed by Alan Grey. The Grey-Markov model has the benefit of not requiring any assumptions, unlike other models, such as having to reach a certain threshold for the amount of data or the patterns of data. Large amounts of data to be obtained in a short length of time, which is challenging. Forecasts based on the Markov chain that can be utilized for predictions that have various time series in a random manner. In the process of developing the Grey Model (1,1), the Markov chain was incorporated, and the resulting model was given the name Grey-Markov Model (1,1). The Grey-Markov model (1,1) makes use of the idea of state transition, in which the state will change from time to time in an environment where the data is unclear. It is the nature of the uncertainty that enables the incorporation of the Markov chain analysis into the Grey model (1,1) [3].

For the purpose of this research, the Grey-Markov model with parameters (1,1) was selected to be applied to the proportion of the population that had recently dealt with health issues. The data used range from the years 1996 to 2021 and are annual data (28 observations). Due to the small amount of accessible data and the fact that it is not an excessively huge amount, the Grey-Markov model (1,1) was chosen for the purpose of predicting the data for the year that would follow.

2. Grey Model (1,1)

Deng established the Grey system theory in 1982. Grey systems theory has developed into a highly powerful paradigm in recent years for addressing the issue of uncertainty in discrete data. The Grey approach is extensively employed in long-term forecasting since it simply takes a little amount of data and a limited number of calculations. The prediction outcomes of the Grey system, however, frequently diverge from the real data in the data graph that suffers significant data volatility [1]. This Grey method is used by many scholars, particularly in the fields of economics, industry, and finance.

2.1. *Accumulated Generating Operation (AGO)*

The real data sequence $x^{(0)}(k)$ is accumulated into the sequence AGO, which is denoted by the notation $x^{(1)}(k)$. Based on Eq. 2.1 [3], time series data or real observation data collected for the study are grouped into a row.

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \tag{2.1}$$

for $k = 1, 2, \ldots, n$ where k is state.
By using the One time-AGO (1-AGO) model in the following equation [4]:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$$

for $k = 1, 2, \ldots, n$ and $x^{(1)}(1) = x^{(0)}(1)$, then the following AGO sequence is generated:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}$$

2.2. *Mean Generating Operation (MGO)*

MGO is the average of the sequences $x^{(1)}(k)$ and $x^{(1)}(k-1)$, which is $z^{(1)}(k)$. The following first order differential equation must be solved in generating the MGO sequence,

$$\frac{dx^{(1)}(1)}{dk} + ax^{(1)}(k) = b \tag{2.2}$$

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{2.3}$$

where

$$z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}$$

for $k = 2, 3, \ldots, n$.
The approach known as the least squares method was applied to determine the values of the parameters a and b. This was accomplished by transforming Eq. 2.3 into the general form of a linear model, which is essentially [5]

$$\mathbf{Y} = \mathbf{X}\beta + \mathbf{e}$$

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(28) \end{bmatrix} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(28) & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} e^{(1)}(2) & 1 \\ e^{(1)}(3) & 1 \\ \vdots & \vdots \\ e^{(1)}(28) & 1 \end{bmatrix}$$

Hence, the MGO sequence can be obtained as described below

$$Z^{(1)} = \{z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n)\} \tag{2.4}$$

2.3. *Grey Model (1,1) Prediction with 1-AGO Process*

$$\mathscr{L}\{\frac{dx^{(1)}(1)}{dk}\} + \mathscr{L}\{ax^{(1)}(k)\} = b$$

$$\mathscr{L}\{x^{(1)}(k)\} = \frac{x^{(1)}(0) + \frac{b}{s}}{s+a}$$

by initial condition, $x^{(1)}(1) = x^{(0)}(1) = x^{(1)}(0)$, and using inverse Laplace transform, then

$$x^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}$$

for $k > 0$.
Then the prediction formula for AGO is [4]

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}$$

After knowing the predicted value of AGO, the remaining step in determining the results of the Grey model (1,1) is the 1-AGO (Inverse Accumulated Generating Operation) procedure, specifically by subtracting the $k-1$ term from the AGO value with the $k-$th term shown in Eq. 2.5 [5]:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{2.5}$$

3. Grey-Markov (1,1) Model

In order to get more precise prediction outcomes with the Grey Model, the Markov chain combining technique is applied in the Grey system (1,1). This particular model is referred to as the Grey-Markov (1,1). After modeling the actual data with the Grey Model (1,1), the next step is to determine the relative error between the value that was predicted and the value that was actually obtained for each of the earlier phases. The Grey-Markov(1,1) model's fundamental premise is that it is feasible to derive potentially accurate prediction values from a Markov transition matrix, which allows for the construction of the transition as well as the relative inaccuracy that is associated with it. The following is an overview of the procedures for the Grey-Markov Model (1,1):

(1) Establish a new data series that will be made up of the predicted values of the Grey model (1,1) using Eq. 2.5, that is

$$\hat{X}^{(0)} = \{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(n)\}$$

where

$$\hat{x}^{(0)}(1) = x^{(0)}(1)$$

(2) The term "state transition" refers to the change in state that occurs when moving from $\hat{x}^{(0)}(1)$ to $\hat{x}^{(0)}(2)$ in a Markov Chain analysis, as well as when moving from $\hat{x}^{(0)}(2)$ to $\hat{x}^{(0)}(n)$ and so on. Before attempting to determine the

state transition, it is necessary to first specify the state that will be employed based on the relative error value (er), as follows [3]:

$$er(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100$$

The next step is to determine relative error intervals for each state, specifically:

$$e(j) = [e(j^-), e(j^+)]$$

where

$$e(j^-) = L + \frac{m-1}{r}(H - L)$$
$$e(j^+) = L + \frac{m}{r}(H - L)$$

L is the smallest relative error, H is the biggest relative error, m is state, and r is the number of state.

(3) Depending on a previously set state interval, defines the state of each piece of data.

(4) Determine the value of the transition's probability by applying the Markovian property stated as

$$P_{ij}(k) = P(X_k = j|X_0 = i) = \frac{n_{ij}(k)}{n_i}$$

where $i = 1, 2, ..., n$ The probability of a transition, denoted by the matrix $P(k)$, can be written as follows in the form of a stochastic matrix:

$$\mathbf{P}(k) = \begin{bmatrix} P_{11}(k) & P_{12}(k) & \dots & P_{1n}(k) \\ P_{21}(k) & P_{22}(k) & \dots & P_{2n}(k) \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1}(k) & P_{n2}(k) & \dots & P_{nn}(k) \end{bmatrix}$$

(5) Determines the number of transitions required for each initial state in each data year used to predict the data

(6) Determines the state in which the forecast year has the highest probability by computing the transition of the state with the highest probability sum value.

(7) Utilizing the following Eq. 3.1, determine the MCGM's value in terms of its ability to forecast the future.

$$\hat{x}(k) = \hat{x}^{(0)}(k)\left(1 + \frac{e(j^-) + e(j^+)}{2} \times \frac{1}{100}\right) \tag{3.1}$$

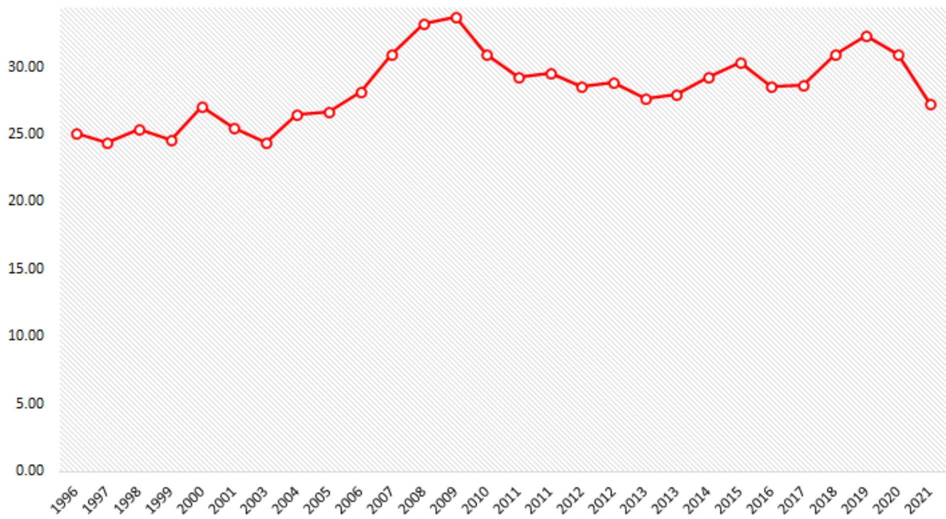
for $k = 1, 2, \dots, n$

4. Case Study

4.1. *Data Descriptive*

As a case study, this study uses statistics on the percentage of the population that experiences health concerns annually. The years 1996 through 2021 are included in the analysis (28 observations). Information collected from the Central Statistics

Agency. A plot of the data from one year to the next is shown in Figure 1. Since there was not a significant amount of variation in the data, as shown in Figure 1, it is considered that the Grey-Markov (1,1) model will be utilized to provide a prediction regarding the percentage of the population that will be experiencing health concerns in 2022.

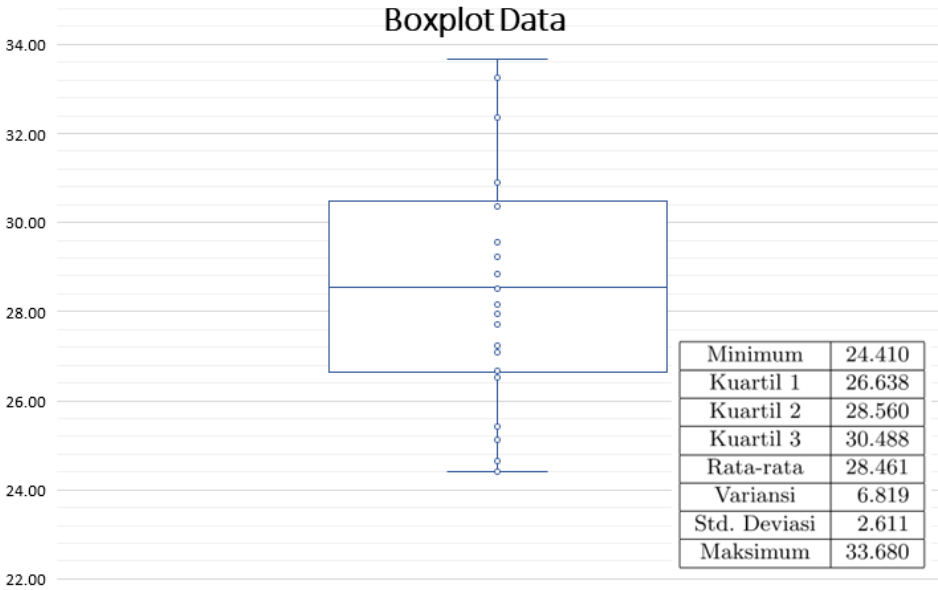


Gambar 1: The Percentage of The Populatioon that Experiences Health Concerns Anually Plot

Boxplots of the data and descriptive statistics of the data are presented in Figure 2, which can be found below. According to the boxplot, there were no noticeable outliers found. In addition, if we take a look at descriptive statistics, we can see that the lowest percentage ever recorded was 24.41 in the year 2002, while the highest percentage ever recorded was 33.68. (occurring in 2009). The Central Statistics Agency (BPS) also stated that the percentage of Indonesians who have health concerns has reduced over the course of the past two years. This information was provided by the BPS. In 2021, it was observed that 27.23% of the population had some sort of health concern during the previous month. According to these numbers, 27 out of every 100 people living in Indonesia had complained about their health in the most recent month. In comparison to the previous year, when it was 30.96%, this year's percentage represents a 3.73 point drop. The arrival of Covid-19 marked the beginning of the downward trend in the number of health concerns. The percentage of the population that reported having health concerns dropped by 1.4 percentage points from 2019's level, which was 32.36%.

4.2. *Data Analysis*

The following is a Grey-Markov (1;1) modeling procedure



Gambar 2: Boxplot of The Data

(1) Defining the AGO sequence, $X^{(1)}(k)$,
Supposing that $X^{(0)}(k)$ is a sequence of observations on the percentage of the population that experienced health complaints one month ago, this would transform into the following:

$$X^{(0)}(k) = \{25,13; 24,41; 25,43; 24,65; 27,08; \ldots; 32,36; 30,96; 27,23\}$$

Furthermore, the AGO sequence formed with

$$X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$$

for $k = 1, 2, \ldots, 28$.
So that,

$$\begin{aligned} X^{(1)}(1) &= 25,13 \\ X^{(1)}(2) &= 25,13 + 24,41 = 49,54 \\ X^{(1)}(3) &= 25,13 + 24,41 + 25,43 = 74,97 \\ &\vdots \\ X^{(1)}(28) &= \cdots + 30,96 + 27,23 = 796,92 \end{aligned}$$

Thus, the sequence 1-AGO formed is

$$X^{(1)}(k) = \{25,13; 49,54; 74,97; 99,62; 126,70; \ldots; 738,72; 769,68; 796,92\}$$

(2) Defining the MGO sequence, $Z^{(1)}(k)$, where

$$Z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}$$

for $k = 2, 3, \ldots, 28$.
So that,

$$\begin{aligned}
Z^{(1)}(2) &= \frac{49,54 - 25,13}{2} = 37,34 \\
Z^{(1)}(3) &= \frac{74,97 - 49,54}{2} = 62,26 \\
Z^{(1)}(4) &= \frac{99,62 - 74,97}{2} = 87,29 \\
&\vdots \\
Z^{(1)}(28) &= \frac{796,92 - 769,68}{2} = 789,30
\end{aligned}$$

Thus, the sequence MGO formed is

$$X^{(1)}(k) = \{37,34; 62,26; 87,29; 113,16; 139,45; \ldots; 722,54; 754,20; 783,30\}$$

(3) Forecasting with 1-AGO, $\hat{X}^{(0)}(k)$,
Note that the general linear model formed is

$$X^{(0)}(k) = -\beta_1 Z^{(1)}(k) + \beta_2 + e^0(k)$$

where

$$\mathbf{X}^{(0)}(k) = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(28) \end{bmatrix}; \mathbf{Z}^{(1)}(k) = \begin{bmatrix} z^{(1)}(2) \\ z^{(1)}(3) \\ \vdots \\ z^{(1)}(28) \end{bmatrix}$$

Furthermore, the estimation of the parameters $beta_1$ and $beta_2$ using the Least Squares approach by constructing a linear model in the equation described by the reference model linear so that the following parameters are obtained:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} -0,006 \\ 26,174 \end{bmatrix}$$

Based on the Eq. 2.5, then the prediction sequence 1-AGO is obtained as follows

$$\hat{X}^0(k) = \{25,13; 26,40; 26,56; 26,73; 26,88; \ldots; 30,55; 30,75; 30,92\}$$

(4) Describes the transitions that occur between states. When determining the transition, the state class is initially determined based on the relative error value of (er), which is then used to determine the transition.

$$e(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \times 100$$

The relative error sequence, **e**, that is obtained is

$$\mathbf{e} = \{0; -8,17; -4,44; -8,42; 0,73; \ldots; 5,59; 0,69; -13,55\}$$

The smallest relative error value is -13.39 (L), while the largest relative error value is 16.23 (H). Next, the relative error intervals ($e(j) = [e(j^-), e(j^+)]$) for (r) situations, with

$$e(j^-) = -13,39 + \frac{k-1}{4}(16,23 - (-13,39))$$
$$e(j^+) = -13,39 + \frac{k}{4}(16,23 - (-13,39))$$

Table 1 shows the relative interval error for 4 states

Tabel 1: The Relative Interval Error for 4 States

State	$e(j^-)$	$e(j^+)$
1	-13.39	-5.99
2	-5.99	1.42
3	1.42	8.82
4	8.82	16.23

In addition, every component of the relative error sequence is classified in accordance with the state class that it belongs to. Because of this, the S sequence includes the state class for each k in the following sequence:

$$\mathbf{S} = \{2; 1; 2; 1; 2; 1; 1; 2; 2; 3; 4; 4; 4; 3; 3; 3; 2; 2; 2; 2; 2; 3; 2; 2; 3; 3; 2; 1\}$$

(5) Calculating the transition probability for each k using Markovian properties, so that the transition probability matrix is obtained as folllows

$$\mathbf{P}_1 = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ \frac{4}{13} & \frac{6}{13} & \frac{3}{13} & 0 \\ 0 & \frac{3}{7} & \frac{3}{7} & \frac{1}{7} \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}; \mathbf{P}_2 = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ \frac{1}{11} & \frac{6}{11} & \frac{3}{11} & \frac{1}{11} \\ \frac{1}{7} & \frac{4}{7} & \frac{1}{7} & \frac{1}{7} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}; \mathbf{P}_3 = \begin{bmatrix} \frac{1}{4} & \frac{2}{4} & \frac{1}{4} & 0 \\ \frac{2}{12} & \frac{6}{12} & \frac{2}{12} & \frac{2}{12} \\ \frac{1}{6} & \frac{3}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 1 & 0 \end{bmatrix}; \mathbf{P}_4 = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{2}{12} & \frac{6}{12} & \frac{2}{12} & \frac{2}{12} \\ 0 & \frac{3}{5} & \frac{2}{5} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}$$

$$\mathbf{P}_5 = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{2}{4} \\ \frac{2}{11} & \frac{3}{11} & \frac{5}{11} & \frac{1}{11} \\ 0 & \frac{4}{5} & \frac{1}{5} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}; \mathbf{P}_6 = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{2}{4} \\ \frac{1}{8} & \frac{4}{8} & \frac{2}{8} & \frac{1}{8} \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \mathbf{P}_7 = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{2}{4} \\ \frac{1}{10} & \frac{3}{10} & \frac{5}{10} & \frac{1}{10} \\ 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \mathbf{P}_8 = \begin{bmatrix} 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{9} & \frac{3}{9} & \frac{3}{9} & \frac{2}{9} \\ 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{P}_9 = \begin{bmatrix} 0 & 0 & \frac{2}{4} & \frac{2}{4} \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \\ 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}; \mathbf{P}_{10} = \begin{bmatrix} 0 & \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \\ \frac{1}{7} & \frac{3}{7} & \frac{1}{7} & \frac{2}{7} \\ 0 & \frac{2}{4} & \frac{2}{4} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}; \mathbf{P}_{11} = \begin{bmatrix} 0 & \frac{2}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{6} & \frac{2}{6} & \frac{2}{6} & \frac{1}{6} \\ 0 & \frac{2}{4} & \frac{2}{4} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}; \mathbf{P}_{12} = \begin{bmatrix} 0 & \frac{2}{4} & \frac{2}{4} & 0 \\ 0 & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{4} & \frac{2}{4} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$$

$$\mathbf{P}_{13}=\begin{bmatrix}0&\frac{2}{4}&\frac{2}{4}&0\\0&\frac{3}{5}&\frac{1}{5}&\frac{1}{5}\\\frac{1}{4}&\frac{1}{4}&\frac{2}{4}&0\\0&\frac{1}{3}&\frac{2}{3}&0\end{bmatrix};\mathbf{P}_{14}=\begin{bmatrix}0&\frac{3}{4}&\frac{1}{4}&0\\0&\frac{3}{5}&\frac{2}{5}&0\\\frac{1}{2}&\frac{1}{2}&0&0\\0&\frac{1}{3}&\frac{2}{3}&0\end{bmatrix};\mathbf{P}_{15}=\begin{bmatrix}0&\frac{3}{4}&\frac{1}{4}&0\\0&\frac{4}{5}&\frac{1}{5}&0\\0&0&1&0\\\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\end{bmatrix};\mathbf{P}_{16}=\begin{bmatrix}0&\frac{3}{4}&\frac{1}{4}&0\\0&\frac{4}{5}&\frac{1}{5}&0\\0&0&1&0\\\frac{1}{2}&\frac{1}{2}&0&0\end{bmatrix}$$

$$\mathbf{P}_{17}=\begin{bmatrix}\frac{1}{5}&\frac{4}{5}&0&0\\0&\frac{2}{5}&\frac{3}{5}&0\\0&1&0&0\\1&0&0&0\end{bmatrix};\mathbf{P}_{18}=\begin{bmatrix}0&\frac{3}{4}&\frac{1}{4}&0\\0&\frac{3}{5}&\frac{2}{5}&0\\\frac{1}{2}&\frac{1}{2}&0&0\\0&\frac{1}{3}&\frac{2}{3}&0\end{bmatrix};\mathbf{P}_{19}=\begin{bmatrix}0&\frac{2}{4}&\frac{2}{4}&0\\0&\frac{4}{5}&\frac{1}{5}&0\\1&0&0&0\\0&0&0&0\end{bmatrix};\mathbf{P}_{20}=\begin{bmatrix}0&\frac{2}{4}&\frac{2}{4}&0\\\frac{1}{4}&\frac{2}{4}&\frac{1}{4}&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$\mathbf{P}_{21}=\begin{bmatrix}\frac{1}{4}&\frac{2}{4}&\frac{1}{4}&0\\0&\frac{1}{3}&\frac{2}{3}&0\\0&0&0&0\\0&0&0&0\end{bmatrix};\mathbf{P}_{22}=\begin{bmatrix}\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\\0&\frac{2}{3}&\frac{1}{3}&0\\0&0&0&0\\0&0&0&0\end{bmatrix};\mathbf{P}_{23}=\begin{bmatrix}0&\frac{1}{2}&\frac{1}{2}&0\\\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\\0&0&0&0\\0&0&0&0\end{bmatrix};\mathbf{P}_{24}=\begin{bmatrix}\frac{1}{2}&0&\frac{1}{2}&0\\0&\frac{1}{2}&\frac{1}{2}&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$\mathbf{P}_{25}=\begin{bmatrix}0&1&0&0\\\frac{1}{2}&0&\frac{1}{2}&0\\0&0&0&0\\0&0&0&0\end{bmatrix};\mathbf{P}_{26}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix};\mathbf{P}_{27}=\begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix};\mathbf{P}_{28}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

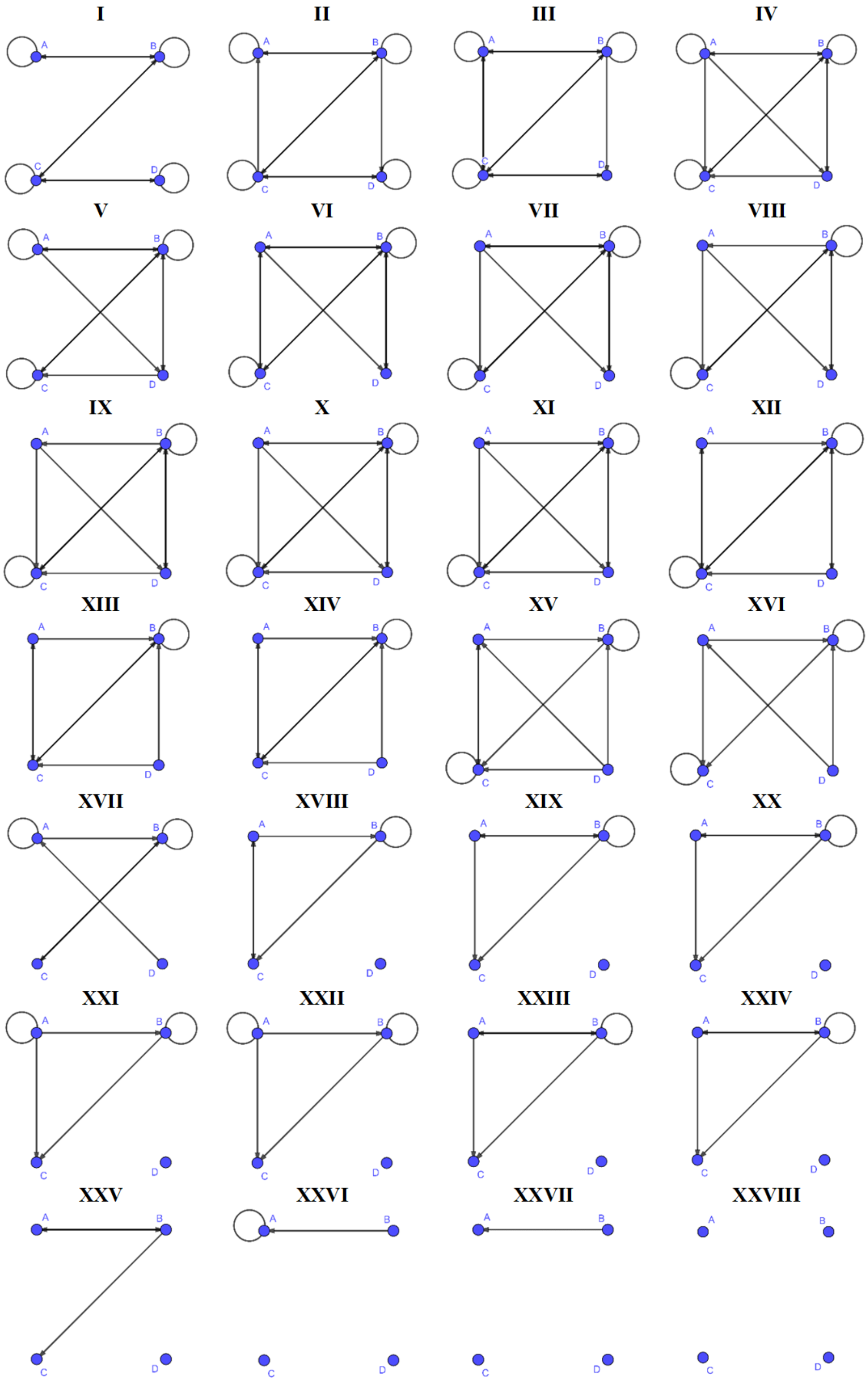
The transition probability matrix can be represented by the following transition diagram

(6) Based on the transition probability matrix, select the state that has the highest possible number of transition probabilities for all states. The outcomes of the calculation used to determine the total number of transitional opportunities are presented in the Table 2.

Tabel 2: State Class based on Relative Error

Initial	Transition (k)	State			
		1	2	3	4
2	28	0	0	0	0
1	27	0	0	0	0
2	26	0	1	0	0
1	25	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮
3	3	0,17	0,50	0,17	0,17
2	2	0,31	0,46	0,23	0
1	1	0,25	0,75	0	0
Total		4,49	11,74	6,39	1,38

The second state class is the one that has the greatest number of transition probabilities of all of the state classes. As a direct consequence of this, the



Gambar 3: Transition Diagram for 28 Transitions

forecast for 2022, based on the value of k, comes into the second state class.

$$e(29) = [-5,99; 1,42]$$

(7) Forecasting for 2022 ($k = 29$),

$$\begin{aligned}\hat{x}(29) &= \hat{x}^{(0)}(29)\Big(1 + \frac{e(29^-) + e(29^+)}{2} \times \frac{1}{100}\Big)\\ &= 31,07\Big(1 + \frac{-5,99 + 1,42}{2} \times \frac{1}{100}\Big)\\ &= 30,36\end{aligned}$$

5. Conclusion

The form of the Grey-Markov (1,1) Model on the percentage of the population experiencing health symptoms is $x(k) = x^{(0)}(k)(1 + \frac{er(j^-)+er(j^+)}{2} \times \frac{1}{100})$ and can be forecasted by the utilization of the Grey-Markov (1,1) Model in its construction. The state or the error value interval in the prediction year will either determine or contribute to the final result of the Grey Markov (1,1) prediction. The results of research on predictions made using the Grey Markov Model (1,1) on the percentage of the population that experiences complaints of health symptoms indicate that the prediction results in 2021 are Rp. 30.36% with an accuracy test rate using a Mean Absolute Percentage Error (MAPE) of 2.43%, which is classified as very accurate. The research on these predictions was conducted in order to determine the percentage of the population that experiences complaints of health symptoms.

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